

1. Introduction

This chapter begins by discussing what statistics are and why the study of statistics is important. Subsequent sections cover a variety of topics all basic to the study of statistics. The only theme common to all of these sections is that they cover concepts and ideas important for other chapters in the book.

- A. What are Statistics?
- B. Importance of Statistics
- C. Descriptive Statistics
- D. Inferential Statistics
- E. Variables
- F. Percentiles
- G. Measurement
- H. Levels of Measurement
- I. Distributions
- J. Summation Notation
- K. Linear Transformations
- L. Logarithms
- M. Exercises

What Are Statistics

by Mikki Hebl

Learning Objectives

1. Describe the range of applications of statistics
2. Identify situations in which statistics can be misleading
3. Define “Statistics”

Statistics include numerical facts and figures. For instance:

- The largest earthquake measured 9.2 on the Richter scale.
- Men are at least 10 times more likely than women to commit murder.
- One in every 8 South Africans is HIV positive.
- By the year 2020, there will be 15 people aged 65 and over for every new baby born.

The study of statistics involves math and relies upon calculations of numbers. But it also relies heavily on how the numbers are chosen and how the statistics are interpreted. For example, consider the following three scenarios and the interpretations based upon the presented statistics. You will find that the numbers may be right, but the interpretation may be wrong. Try to identify a major flaw with each interpretation before we describe it.

1) A new advertisement for Ben and Jerry's ice cream introduced in late May of last year resulted in a 30% increase in ice cream sales for the following three months. Thus, the advertisement was effective.

A major flaw is that ice cream consumption generally increases in the months of June, July, and August regardless of advertisements. This effect is called a history effect and leads people to interpret outcomes as the result of one variable when another variable (in this case, one having to do with the passage of time) is actually responsible.

2) The more churches in a city, the more crime there is. Thus, churches lead to crime.

A major flaw is that both increased churches and increased crime rates can be explained by larger populations. In bigger cities, there are both more churches and more crime. This problem, which we will discuss in more detail in Chapter 6, refers to the third-variable problem. Namely, a third variable can cause both situations; however, people erroneously believe that there is a causal relationship between the two primary variables rather than recognize that a third variable can cause both.

3) 75% more interracial marriages are occurring this year than 25 years ago. Thus, our society accepts interracial marriages.

A major flaw is that we don't have the information that we need. What is the rate at which marriages are occurring? Suppose only 1% of marriages 25 years ago were interracial and so now 1.75% of marriages are interracial (1.75 is 75% higher than 1). But this latter number is hardly evidence suggesting the acceptability of interracial marriages. In addition, the statistic provided does not rule out the possibility that the number of interracial marriages has seen dramatic fluctuations over the years and this year is not the highest. Again, there is simply not enough information to understand fully the impact of the statistics.

As a whole, these examples show that statistics are *not only facts and figures*; they are something more than that. In the broadest sense, “statistics” refers to a range of techniques and procedures for analyzing, interpreting, displaying, and making decisions based on data.

Importance of Statistics

by Mikki Hebl

Learning Objectives

1. Give examples of statistics encountered in everyday life
2. Give examples of how statistics can lend credibility to an argument

Like most people, you probably feel that it is important to “take control of your life.” But what does this mean? Partly, it means being able to properly evaluate the data and claims that bombard you every day. If you cannot distinguish good from faulty reasoning, then you are vulnerable to manipulation and to decisions that are not in your best interest. Statistics provides tools that you need in order to react intelligently to information you hear or read. In this sense, statistics is one of the most important things that you can study.

To be more specific, here are some claims that we have heard on several occasions. (We are not saying that each one of these claims is true!)

- 4 out of 5 dentists recommend Dentine.
- Almost 85% of lung cancers in men and 45% in women are tobacco-related.
- Condoms are effective 94% of the time.
- Native Americans are significantly more likely to be hit crossing the street than are people of other ethnicities.
- People tend to be more persuasive when they look others directly in the eye and speak loudly and quickly.
- Women make 75 cents to every dollar a man makes when they work the same job.
- A surprising new study shows that eating egg whites can increase one's life span.
- People predict that it is very unlikely there will ever be another baseball player with a batting average over 400.
- There is an 80% chance that in a room full of 30 people that at least two people will share the same birthday.
- 79.48% of all statistics are made up on the spot.

All of these claims are statistical in character. We suspect that some of them sound familiar; if not, we bet that you have heard other claims like them. Notice how diverse the examples are. They come from psychology, health, law, sports, business, etc. Indeed, data and data interpretation show up in discourse from virtually every facet of contemporary life.

Statistics are often presented in an effort to add credibility to an argument or advice. You can see this by paying attention to television advertisements. Many of the numbers thrown about in this way do not represent careful statistical analysis. They can be misleading and push you into decisions that you might find cause to regret. For these reasons, learning about statistics is a long step towards taking control of your life. (It is not, of course, the only step needed for this purpose.) The present electronic textbook is designed to help you learn statistical essentials. **It will make you into an intelligent consumer of statistical claims.**

You can take the first step right away. To be an intelligent consumer of statistics, your first reflex must be to **question** the statistics that you encounter. The British Prime Minister Benjamin Disraeli is quoted by Mark Twain as having said, "There are three kinds of lies -- lies, damned lies, and statistics." This quote reminds us why it is so important to understand statistics. So let us invite you to reform your statistical habits from now on. No longer will you blindly accept numbers or findings. Instead, you will begin to think about the numbers, their sources, and most importantly, the procedures used to generate them.

We have put the emphasis on defending ourselves against fraudulent claims wrapped up as statistics. We close this section on a more positive note. Just as important as detecting the deceptive use of statistics is the appreciation of the proper use of statistics. You must also learn to recognize statistical evidence that supports a stated conclusion. Statistics are all around you, sometimes used well, sometimes not. We must learn how to distinguish the two cases.

Now let us get to work!

Descriptive Statistics

by Mikki Hebl

Prerequisites

- none

Learning Objectives

1. Define “descriptive statistics”
2. Distinguish between descriptive statistics and inferential statistics

Descriptive statistics are numbers that are used to summarize and describe data. The word “data” refers to the information that has been collected from an experiment, a survey, an historical record, etc. (By the way, “data” is plural. One piece of information is called a “datum.”) If we are analyzing birth certificates, for example, a descriptive statistic might be the percentage of certificates issued in New York State, or the average age of the mother. Any other number we choose to compute also counts as a descriptive statistic for the data from which the statistic is computed. Several descriptive statistics are often used at one time to give a full picture of the data.

Descriptive statistics are just descriptive. They do not involve **generalizing** beyond the data at hand. Generalizing from our data to another set of cases is the business of inferential statistics, which you'll be studying in another section. Here we focus on (mere) descriptive statistics.

Some descriptive statistics are shown in Table 1. The table shows the average salaries for various occupations in the United States in 1999.

Table 1. Average salaries for various occupations in 1999.

\$112,760	pediatricians
\$106,130	dentists
\$100,090	podiatrists
\$76,140	physicists
\$53,410	architects,
\$49,720	school, clinical, and counseling psychologists
\$47,910	flight attendants
\$39,560	elementary school teachers
\$38,710	police officers
\$18,980	floral designers

Descriptive statistics like these offer insight into American society. It is interesting to note, for example, that we pay the people who educate our children and who protect our citizens a great deal less than we pay people who take care of our feet or our teeth.

For more descriptive statistics, consider Table 2. It shows the number of unmarried men per 100 unmarried women in U.S. Metro Areas in 1990. From this table we see that men outnumber women most in Jacksonville, NC, and women outnumber men most in Sarasota, FL. You can see that descriptive statistics can be useful if we are looking for an opposite-sex partner! (These data come from the Information Please Almanac.)

Table 2. Number of unmarried men per 100 unmarried women in U.S. Metro Areas in 1990.

Cities with mostly men	Men per 100 Women	Cities with mostly women	Men per 100 Women
1. Jacksonville, NC	224	1. Sarasota, FL	66
2. Killeen-Temple, TX	123	2. Bradenton, FL	68
3. Fayetteville, NC	118	3. Altoona, PA	69

4. Brazoria, TX	117	4. Springfield, IL	70
5. Lawton, OK	116	5. Jacksonville, TN	70
6. State College, PA	113	6. Gadsden, AL	70
7. Clarksville- Hopkinsville, TN-KY	113	7. Wheeling, WV	70
8. Anchorage, Alaska	112	8. Charleston, WV	71
9. Salinas-Seaside- Monterey, CA	112	9. St. Joseph, MO	71
10. Bryan-College Station, TX	111	10. Lynchburg, VA	71

NOTE: Unmarried includes never-married, widowed, and divorced persons, 15 years or older.

These descriptive statistics may make us ponder why the numbers are so disparate in these cities. One potential explanation, for instance, as to why there are more women in Florida than men may involve the fact that elderly individuals tend to move down to the Sarasota region and that women tend to outlive men. Thus, more women might live in Sarasota than men. However, in the absence of proper data, this is only speculation.

You probably know that descriptive statistics are central to the world of sports. Every sporting event produces numerous statistics such as the shooting percentage of players on a basketball team. For the Olympic marathon (a foot race of 26.2 miles), we possess data that cover more than a century of competition. (The first modern Olympics took place in 1896.) The following table shows the winning times for both men and women (the latter have only been allowed to compete since 1984).

Table 3. Winning Olympic marathon times.

Women			
Year	Winner	Country	Time
1984	Joan Benoit	USA	2:24:52
1988	Rosa Mota	POR	2:25:40

1992	Valentina Yegorova	UT	2:32:41
1996	Fatuma Roba	ETH	2:26:05
2000	Naoko Takahashi	JPN	2:23:14
2004	Mizuki Noguchi	JPN	2:26:20
Men			
Year	Winner	Country	Time
1896	Spiridon Louis	GRE	2:58:50
1900	Michel Theato	FRA	2:59:45
1904	Thomas Hicks	USA	3:28:53
1906	Billy Sherring	CAN	2:51:23
1908	Johnny Hayes	USA	2:55:18
1912	Kenneth McArthur	S. Afr.	2:36:54
1920	Hannes Kolehmainen	FIN	2:32:35
1924	Albin Stenroos	FIN	2:41:22
1928	Boughra El Ouafi	FRA	2:32:57
1932	Juan Carlos Zabala	ARG	2:31:36
1936	Sohn Kee-Chung	JPN	2:29:19
1948	Delfo Cabrera	ARG	2:34:51
1952	Emil Ztopek	CZE	2:23:03
1956	Alain Mimoun	FRA	2:25:00
1960	Abebe Bikila	ETH	2:15:16
1964	Abebe Bikila	ETH	2:12:11
1968	Mamo Wolde	ETH	2:20:26
1972	Frank Shorter	USA	2:12:19
1976	Waldemar Cierpinski	E.Ger	2:09:55
1980	Waldemar Cierpinski	E.Ger	2:11:03
1984	Carlos Lopes	POR	2:09:21
1988	Gelindo Bordin	ITA	2:10:32

1992	Hwang Young-Cho	S. Kor	2:13:23
1996	Josia Thugwane	S. Afr.	2:12:36
2000	Gezahenge Abera	ETH	2:10:10
2004	Stefano Baldini	ITA	2:10:55

There are many descriptive statistics that we can compute from the data in the table. To gain insight into the improvement in speed over the years, let us divide the men's times into two pieces, namely, the first 13 races (up to 1952) and the second 13 (starting from 1956). The mean winning time for the first 13 races is 2 hours, 44 minutes, and 22 seconds (written 2:44:22). The mean winning time for the second 13 races is 2:13:18. This is quite a difference (over half an hour). Does this prove that the fastest men are running faster? Or is the difference just due to chance, no more than what often emerges from chance differences in performance from year to year? We can't answer this question with descriptive statistics alone. All we can affirm is that the two means are “suggestive.”

Examining Table 3 leads to many other questions. We note that Takahashi (the lead female runner in 2000) would have beaten the male runner in 1956 and all male runners in the first 12 marathons. This fact leads us to ask whether the gender gap will close or remain constant. When we look at the times within each gender, we also wonder how far they will decrease (if at all) in the next century of the Olympics. Might we one day witness a sub-2 hour marathon? The study of statistics can help you make reasonable guesses about the answers to these questions.

Inferential Statistics

by Mikki Hebl

Prerequisites

- Chapter 1: Descriptive Statistics

Learning Objectives

1. Distinguish between a sample and a population
2. Define inferential statistics
3. Identify biased samples
4. Distinguish between simple random sampling and stratified sampling
5. Distinguish between random sampling and random assignment

Populations and samples

In statistics, we often rely on a sample --- that is, a small subset of a larger set of data --- to draw inferences about the larger set. The larger set is known as the population from which the sample is drawn.

Example #1: You have been hired by the National Election Commission to examine how the American people feel about the fairness of the voting procedures in the U.S. Who will you ask?

It is not practical to ask every single American how he or she feels about the fairness of the voting procedures. Instead, we query a relatively small number of Americans, and draw inferences about the entire country from their responses. The Americans actually queried constitute our sample of the larger population of all Americans. The mathematical procedures whereby we convert information about the sample into intelligent guesses about the population fall under the rubric of inferential statistics.

A sample is typically a small subset of the population. In the case of voting attitudes, we would sample a few thousand Americans drawn from the hundreds of millions that make up the country. In choosing a sample, it is therefore crucial that it not over-represent one kind of citizen at the expense of others. For example, something would be wrong with our sample if it happened to be made up entirely of Florida residents. If the sample held only Floridians, it could not be used to infer

the attitudes of other Americans. The same problem would arise if the sample were comprised only of Republicans. Inferential statistics are based on the assumption that sampling is random. We trust a random sample to represent different segments of society in close to the appropriate proportions (provided the sample is large enough; see below).

Example #2: We are interested in examining how many math classes have been taken on average by current graduating seniors at American colleges and universities during their four years in school. Whereas our population in the last example included all US citizens, now it involves just the graduating seniors throughout the country. This is still a large set since there are thousands of colleges and universities, each enrolling many students. (New York University, for example, enrolls 48,000 students.) It would be prohibitively costly to examine the transcript of every college senior. We therefore take a sample of college seniors and then make inferences to the entire population based on what we find. To make the sample, we might first choose some public and private colleges and universities across the United States. Then we might sample 50 students from each of these institutions. Suppose that the average number of math classes taken by the people in our sample were 3.2. Then we might speculate that 3.2 approximates the number we would find if we had the resources to examine every senior in the entire population. But we must be careful about the possibility that our sample is non-representative of the population. Perhaps we chose an overabundance of math majors, or chose too many technical institutions that have heavy math requirements. Such bad sampling makes our sample unrepresentative of the population of all seniors.

To solidify your understanding of sampling bias, consider the following example. Try to identify the population and the sample, and then reflect on whether the sample is likely to yield the information desired.

Example #3: A substitute teacher wants to know how students in the class did on their last test. The teacher asks the 10 students sitting in the front row to state their latest test score. He concludes from their report that the class did extremely well. What is the sample? What is the population? Can you identify any problems with choosing the sample in the way that the teacher did?

In Example #3, the population consists of all students in the class. The sample is made up of just the 10 students sitting in the front row. The sample is not likely to be representative of the population. Those who sit in the front row tend to be more interested in the class and tend to perform higher on tests. Hence, the sample may perform at a higher level than the population.

Example #4: A coach is interested in how many cartwheels the average college freshmen at his university can do. Eight volunteers from the freshman class step forward. After observing their performance, the coach concludes that college freshmen can do an average of 16 cartwheels in a row without stopping.

In Example #4, the population is the class of all freshmen at the coach's university. The sample is composed of the 8 volunteers. The sample is poorly chosen because volunteers are more likely to be able to do cartwheels than the average freshman; people who can't do cartwheels probably did not volunteer! In the example, we are also not told of the gender of the volunteers. Were they all women, for example? That might affect the outcome, contributing to the non-representative nature of the sample (if the school is co-ed).

Simple Random Sampling

Researchers adopt a variety of sampling strategies. The most straightforward is simple random sampling. Such sampling requires every member of the population to have an equal chance of being selected into the sample. In addition, the selection of one member must be independent of the selection of every other member. That is, picking one member from the population must not increase or decrease the probability of picking any other member (relative to the others). In this sense, we can say that simple random sampling chooses a sample by pure chance. To check

your understanding of simple random sampling, consider the following example. What is the population? What is the sample? Was the sample picked by simple random sampling? Is it biased?

Example #5: A research scientist is interested in studying the experiences of twins raised together versus those raised apart. She obtains a list of twins from the **National Twin Registry**, and selects two subsets of individuals for her study. First, she chooses all those in the registry whose last name begins with Z. Then she turns to all those whose last name begins with B. Because there are so many names that start with B, however, our researcher decides to incorporate only every other name into her sample. Finally, she mails out a survey and compares characteristics of twins raised apart versus together.

In Example #5, the population consists of all twins recorded in the National Twin Registry. It is important that the researcher only make statistical generalizations to the twins on this list, not to all twins in the nation or world. That is, the National Twin Registry may not be representative of all twins. Even if inferences are limited to the Registry, a number of problems affect the sampling procedure we described. For instance, choosing only twins whose last names begin with Z does not give every individual an equal chance of being selected into the sample. Moreover, such a procedure risks over-representing ethnic groups with many surnames that begin with Z. There are other reasons why choosing just the Z's may bias the sample. Perhaps such people are more patient than average because they often find themselves at the end of the line! The same problem occurs with choosing twins whose last name begins with B. An additional problem for the B's is that the "every-other-one" procedure disallowed adjacent names on the B part of the list from being both selected. Just this defect alone means the sample was not formed through simple random sampling.

Sample size matters

Recall that the definition of a random sample is a sample in which every member of the population has an equal chance of being selected. This means that the **sampling procedure** rather than the **results** of the procedure define what it means for a sample to be random. Random samples, especially if the sample size is small,

are not necessarily representative of the entire population. For example, if a random sample of 20 subjects were taken from a population with an equal number of males and females, there would be a nontrivial probability (0.06) that 70% or more of the sample would be female. (To see how to obtain this probability, see the section on the binomial distribution in Chapter 5.) Such a sample would not be representative, although it would be drawn randomly. Only a large sample size makes it likely that our sample is close to representative of the population. For this reason, inferential statistics take into account the sample size when generalizing results from samples to populations. In later chapters, you'll see what kinds of mathematical techniques ensure this sensitivity to sample size.

More complex sampling

Sometimes it is not feasible to build a sample using simple random sampling. To see the problem, consider the fact that both Dallas and Houston are competing to be hosts of the 2012 Olympics. Imagine that you are hired to assess whether most Texans prefer Houston to Dallas as the host, or the reverse. Given the impracticality of obtaining the opinion of every single Texan, you must construct a sample of the Texas population. But now notice how difficult it would be to proceed by simple random sampling. For example, how will you contact those individuals who don't vote and don't have a phone? Even among people you find in the telephone book, how can you identify those who have just relocated to California (and had no reason to inform you of their move)? What do you do about the fact that since the beginning of the study, an additional 4,212 people took up residence in the state of Texas? As you can see, it is sometimes very difficult to develop a truly random procedure. For this reason, other kinds of sampling techniques have been devised. We now discuss two of them.

Random assignment

In experimental research, populations are often hypothetical. For example, in an experiment comparing the effectiveness of a new anti-depressant drug with a placebo, there is no actual population of individuals taking the drug. In this case, a specified population of people with some degree of depression is defined and a random sample is taken from this population. The sample is then randomly divided into two groups; one group is assigned to the treatment condition (drug) and the other group is assigned to the control condition (placebo). This random division of

the sample into two groups is called **random assignment**. Random assignment is critical for the validity of an experiment. For example, consider the bias that could be introduced if the first 20 subjects to show up at the experiment were assigned to the experimental group and the second 20 subjects were assigned to the control group. It is possible that subjects who show up late tend to be more depressed than those who show up early, thus making the experimental group less depressed than the control group even before the treatment was administered.

In experimental research of this kind, failure to assign subjects randomly to groups is generally more serious than having a non-random sample. Failure to randomize (the former error) invalidates the experimental findings. A non-random sample (the latter error) simply restricts the generalizability of the results.

Stratified Sampling

Since simple random sampling often does not ensure a representative sample, a sampling method called stratified random sampling is sometimes used to make the sample more representative of the population. This method can be used if the population has a number of distinct “strata” or groups. In stratified sampling, you first identify members of your sample who belong to each group. Then you randomly sample from each of those subgroups in such a way that the sizes of the subgroups in the sample are proportional to their sizes in the population.

Let's take an example: Suppose you were interested in views of capital punishment at an urban university. You have the time and resources to interview 200 students. The student body is diverse with respect to age; many older people work during the day and enroll in night courses (average age is 39), while younger students generally enroll in day classes (average age of 19). It is possible that night students have different views about capital punishment than day students. If 70% of the students were day students, it makes sense to ensure that 70% of the sample consisted of day students. Thus, your sample of 200 students would consist of 140 day students and 60 night students. The proportion of day students in the sample and in the population (the entire university) would be the same. Inferences to the entire population of students at the university would therefore be more secure.

Variables

by Heidi Ziemer

Prerequisites

- none

Learning Objectives

1. Define and distinguish between independent and dependent variables
2. Define and distinguish between discrete and continuous variables
3. Define and distinguish between qualitative and quantitative variables

Independent and dependent variables

Variables are properties or characteristics of some event, object, or person that can take on different values or amounts (as opposed to constants such as π that do not vary). When conducting research, experimenters often manipulate variables. For example, an experimenter might compare the effectiveness of four types of antidepressants. In this case, the variable is “type of antidepressant.” When a variable is manipulated by an experimenter, it is called an independent variable. The experiment seeks to determine the effect of the independent variable on relief from depression. In this example, relief from depression is called a dependent variable. In general, the independent variable is manipulated by the experimenter and its effects on the dependent variable are measured.

Example #1: Can blueberries slow down aging? A study indicates that antioxidants found in blueberries may slow down the process of aging. In this study, 19-month-old rats (equivalent to 60-year-old humans) were fed either their standard diet or a diet supplemented by either blueberry, strawberry, or spinach powder. After eight weeks, the rats were given memory and motor skills tests. Although all supplemented rats showed improvement, those supplemented with blueberry powder showed the most notable improvement.

1. What is the independent variable? (dietary supplement: none, blueberry, strawberry, and spinach)

2. What are the dependent variables? (memory test and motor skills test)

Example #2: Does beta-carotene protect against cancer? Beta-carotene supplements have been thought to protect against cancer. However, a study published in the Journal of the National Cancer Institute suggests this is false. The study was conducted with 39,000 women aged 45 and up. These women were randomly assigned to receive a beta-carotene supplement or a placebo, and their health was studied over their lifetime. Cancer rates for women taking the beta-carotene supplement did not differ systematically from the cancer rates of those women taking the placebo.

1. What is the independent variable? (supplements: beta-carotene or placebo)

2. What is the dependent variable? (occurrence of cancer)

Example #3: How bright is right? An automobile manufacturer wants to know how bright brake lights should be in order to minimize the time required for the driver of a following car to realize that the car in front is stopping and to hit the brakes.

1. What is the independent variable? (brightness of brake lights)

2. What is the dependent variable? (time to hit brakes)

Levels of an Independent Variable

If an experiment compares an experimental treatment with a control treatment, then the independent variable (type of treatment) has two levels: experimental and control. If an experiment were comparing five types of diets, then the independent variable (type of diet) would have 5 levels. In general, the number of levels of an independent variable is the number of experimental conditions.

Qualitative and Quantitative Variables

An important distinction between variables is between qualitative variables and quantitative variables. Qualitative variables are those that express a qualitative attribute such as hair color, eye color, religion, favorite movie, gender, and so on. The values of a qualitative variable do not imply a numerical ordering. Values of the variable “religion” differ qualitatively; no ordering of religions is implied. Qualitative variables are sometimes referred to as categorical variables. Quantitative variables are those variables that are measured in terms of numbers. Some examples of quantitative variables are height, weight, and shoe size.

In the study on the effect of diet discussed previously, the independent variable was type of supplement: none, strawberry, blueberry, and spinach. The variable “type of supplement” is a qualitative variable; there is nothing quantitative about it. In contrast, the dependent variable “memory test” is a quantitative variable since memory performance was measured on a quantitative scale (number correct).

Discrete and Continuous Variables

Variables such as number of children in a household are called discrete variables since the possible scores are discrete points on the scale. For example, a household could have three children or six children, but not 4.53 children. Other variables such as “time to respond to a question” are continuous variables since the scale is continuous and not made up of discrete steps. The response time could be 1.64 seconds, or it could be 1.64237123922121 seconds. Of course, the practicalities of measurement preclude most measured variables from being truly continuous.

Percentiles

by David Lane

Prerequisites

- none

Learning Objectives

1. Define percentiles
2. Use three formulas for computing percentiles

A test score in and of itself is usually difficult to interpret. For example, if you learned that your score on a measure of shyness was 35 out of a possible 50, you would have little idea how shy you are compared to other people. More relevant is the percentage of people with lower shyness scores than yours. This percentage is called a percentile. If 65% of the scores were below yours, then your score would be the 65th percentile.

Two Simple Definitions of Percentile

There is no universally accepted definition of a percentile. Using the 65th percentile as an example, the 65th percentile can be defined as the lowest score that is greater than 65% of the scores. This is the way we defined it above and we will call this “Definition 1.” The 65th percentile can also be defined as the smallest score that is greater than or equal to 65% of the scores. This we will call “Definition 2.” Unfortunately, these two definitions can lead to dramatically different results, especially when there is relatively little data. Moreover, neither of these definitions is explicit about how to handle rounding. For instance, what rank is required to be higher than 65% of the scores when the total number of scores is 50? This is tricky because 65% of 50 is 32.5. How do we find the lowest number that is higher than 32.5% of the scores? A third way to compute percentiles (presented below) is a weighted average of the percentiles computed according to the first two definitions. This third definition handles rounding more gracefully than the other two and has the advantage that it allows the median to be defined conveniently as the 50th percentile.

A Third Definition

Unless otherwise specified, when we refer to “percentile,” we will be referring to this third definition of percentiles. Let's begin with an example. Consider the 25th percentile for the 8 numbers in Table 1. Notice the numbers are given ranks ranging from 1 for the lowest number to 8 for the highest number.

Table 1. Test Scores.

Number	Rank
3	1
5	2
7	3
8	4
9	5
11	6
13	7
15	8

The first step is to compute the rank (R) of the 25th percentile. This is done using the following formula:

$$R = \frac{P}{100} \times (N + 1)$$

where P is the desired percentile (25 in this case) and N is the number of numbers (8 in this case). Therefore,

$$R = \frac{25}{100} \times (8 + 1) = \frac{9}{4} = 2.25$$

If R is an integer, the Pth percentile is be the number with rank R. When R is not an integer, we compute the Pth percentile by interpolation as follows:

1. Define IR as the integer portion of R (the number to the left of the decimal point). For this example, IR = 2.
2. Define FR as the fractional portion of R. For this example, FR = 0.25.

3. Find the scores with Rank I_R and with Rank $I_R + 1$. For this example, this means the score with Rank 2 and the score with Rank 3. The scores are 5 and 7.
4. Interpolate by multiplying the difference between the scores by F_R and add the result to the lower score. For these data, this is $(0.25)(7 - 5) + 5 = 5.5$.

Therefore, the 25th percentile is 5.5. If we had used the first definition (the smallest score greater than 25% of the scores), the 25th percentile would have been 7. If we had used the second definition (the smallest score greater than or equal to 25% of the scores), the 25th percentile would have been 5.

For a second example, consider the 20 quiz scores shown in Table 2.

Table 2. 20 Quiz Scores.

Score	Rank
4	1
4	2
5	3
5	4
5	5
5	6
6	7
6	8
6	9
7	10
7	11
7	12
8	13
8	14
9	15
9	16
9	17
10	18
10	19
10	20

We will compute the 25th and the 85th percentiles. For the 25th,

$$R = \frac{25}{100} \times (20 + 1) = \frac{21}{4} = 5.25$$

$$IR = 5 \text{ and } FR = 0.25.$$

Since the score with a rank of IR (which is 5) and the score with a rank of IR + 1 (which is 6) are both equal to 5, the 25th percentile is 5. In terms of the formula:

$$25\text{th percentile} = (.25) \times (5 - 5) + 5 = 5.$$

For the 85th percentile,

$$R = \frac{85}{100} \times (20 + 1) = 17.85$$

$$IR = 17 \text{ and } FR = 0.85$$

Caution: FR does not generally equal the percentile to be computed as it does here.

The score with a rank of 17 is 9 and the score with a rank of 18 is 10. Therefore, the 85th percentile is:

$$(0.85)(10 - 9) + 9 = 9.85$$

Consider the 50th percentile of the numbers 2, 3, 5, 9.

$$R = \frac{50}{100} \times (4 + 1) = 2.5$$

$$IR = 2 \text{ and } FR = 0.5.$$

The score with a rank of IR is 3 and the score with a rank of IR + 1 is 5. Therefore, the 50th percentile is:

$$(0.5)(5 - 3) + 3 = 4.$$

Finally, consider the 50th percentile of the numbers 2, 3, 5, 9, 11.

$$R = \frac{50}{100} \times (5 + 1) = 3$$

$$IR = 3 \text{ and } FR = 0.$$

Whenever $FR = 0$, you simply find the number with rank IR . In this case, the third number is equal to 5, so the 50th percentile is 5. You will also get the right answer if you apply the general formula:

$$50th\ percentile = (0.00) (9 - 5) + 5 = 5.$$

Levels of Measurement

by Dan Osherson and David M. Lane

Prerequisites

- Chapter 1: Variables

Learning Objectives

1. Define and distinguish among nominal, ordinal, interval, and ratio scales
2. Identify a scale type
3. Discuss the type of scale used in psychological measurement
4. Give examples of errors that can be made by failing to understand the proper use of measurement scales

Types of Scales

Before we can conduct a statistical analysis, we need to measure our dependent variable. Exactly how the measurement is carried out depends on the type of variable involved in the analysis. Different types are measured differently. To measure the time taken to respond to a stimulus, you might use a stop watch. Stop watches are of no use, of course, when it comes to measuring someone's attitude towards a political candidate. A rating scale is more appropriate in this case (with labels like “very favorable,” “somewhat favorable,” etc.). For a dependent variable such as “favorite color,” you can simply note the color-word (like “red”) that the subject offers.

Although procedures for measurement differ in many ways, they can be classified using a few fundamental categories. In a given category, all of the procedures share some properties that are important for you to know about. The categories are called “scale types,” or just “scales,” and are described in this section.

Nominal scales

When measuring using a nominal scale, one simply names or categorizes responses. Gender, handedness, favorite color, and religion are examples of variables measured on a nominal scale. The essential point about nominal scales is that they do not imply any ordering among the responses. For example, when classifying people according to their favorite color, there is no sense in which

green is placed “ahead of” blue. Responses are merely categorized. Nominal scales embody the lowest level of measurement.

Ordinal scales

A researcher wishing to measure consumers' satisfaction with their microwave ovens might ask them to specify their feelings as either “very dissatisfied,” “somewhat dissatisfied,” “somewhat satisfied,” or “very satisfied.” The items in this scale are ordered, ranging from least to most satisfied. This is what distinguishes ordinal from nominal scales. Unlike nominal scales, ordinal scales allow comparisons of the degree to which two subjects possess the dependent variable. For example, our satisfaction ordering makes it meaningful to assert that one person is more satisfied than another with their microwave ovens. Such an assertion reflects the first person's use of a verbal label that comes later in the list than the label chosen by the second person.

On the other hand, ordinal scales fail to capture important information that will be present in the other scales we examine. In particular, the difference between two levels of an ordinal scale cannot be assumed to be the same as the difference between two other levels. In our satisfaction scale, for example, the difference between the responses “very dissatisfied” and “somewhat dissatisfied” is probably not equivalent to the difference between “somewhat dissatisfied” and “somewhat satisfied.” Nothing in our measurement procedure allows us to determine whether the two differences reflect the same difference in psychological satisfaction. Statisticians express this point by saying that the differences between adjacent scale values do not necessarily represent equal intervals on the underlying scale giving rise to the measurements. (In our case, the underlying scale is the true feeling of satisfaction, which we are trying to measure.)

What if the researcher had measured satisfaction by asking consumers to indicate their level of satisfaction by choosing a number from one to four? Would the difference between the responses of one and two necessarily reflect the same difference in satisfaction as the difference between the responses two and three? The answer is No. Changing the response format to numbers does not change the meaning of the scale. We still are in no position to assert that the mental step from 1 to 2 (for example) is the same as the mental step from 3 to 4.

Interval scales

Interval scales are numerical scales in which intervals have the same interpretation throughout. As an example, consider the Fahrenheit scale of temperature. The difference between 30 degrees and 40 degrees represents the same temperature difference as the difference between 80 degrees and 90 degrees. This is because each 10-degree interval has the same physical meaning (in terms of the kinetic energy of molecules).

Interval scales are not perfect, however. In particular, they do not have a true zero point even if one of the scaled values happens to carry the name “zero.” The Fahrenheit scale illustrates the issue. Zero degrees Fahrenheit does not represent the complete absence of temperature (the absence of any molecular kinetic energy). In reality, the label “zero” is applied to its temperature for quite accidental reasons connected to the history of temperature measurement. Since an interval scale has no true zero point, it does not make sense to compute ratios of temperatures. For example, there is no sense in which the ratio of 40 to 20 degrees Fahrenheit is the same as the ratio of 100 to 50 degrees; no interesting physical property is preserved across the two ratios. After all, if the “zero” label were applied at the temperature that Fahrenheit happens to label as 10 degrees, the two ratios would instead be 30 to 10 and 90 to 40, no longer the same! For this reason, it does not make sense to say that 80 degrees is “twice as hot” as 40 degrees. Such a claim would depend on an arbitrary decision about where to “start” the temperature scale, namely, what temperature to call zero (whereas the claim is intended to make a more fundamental assertion about the underlying physical reality).

Ratio scales

The ratio scale of measurement is the most informative scale. It is an interval scale with the additional property that its zero position indicates the absence of the quantity being measured. You can think of a ratio scale as the three earlier scales rolled up in one. Like a nominal scale, it provides a name or category for each object (the numbers serve as labels). Like an ordinal scale, the objects are ordered (in terms of the ordering of the numbers). Like an interval scale, the same difference at two places on the scale has the same meaning. And in addition, the same ratio at two places on the scale also carries the same meaning.

The Fahrenheit scale for temperature has an arbitrary zero point and is therefore not a ratio scale. However, zero on the Kelvin scale is absolute zero. This

makes the Kelvin scale a ratio scale. For example, if one temperature is twice as high as another as measured on the Kelvin scale, then it has twice the kinetic energy of the other temperature.

Another example of a ratio scale is the amount of money you have in your pocket right now (25 cents, 55 cents, etc.). Money is measured on a ratio scale because, in addition to having the properties of an interval scale, it has a true zero point: if you have zero money, this implies the absence of money. Since money has a true zero point, it makes sense to say that someone with 50 cents has twice as much money as someone with 25 cents (or that Bill Gates has a million times more money than you do).

What level of measurement is used for psychological variables?

Rating scales are used frequently in psychological research. For example, experimental subjects may be asked to rate their level of pain, how much they like a consumer product, their attitudes about capital punishment, their confidence in an answer to a test question. Typically these ratings are made on a 5-point or a 7-point scale. These scales are ordinal scales since there is no assurance that a given difference represents the same thing across the range of the scale. For example, there is no way to be sure that a treatment that reduces pain from a rated pain level of 3 to a rated pain level of 2 represents the same level of relief as a treatment that reduces pain from a rated pain level of 7 to a rated pain level of 6.

In memory experiments, the dependent variable is often the number of items correctly recalled. What scale of measurement is this? You could reasonably argue that it is a ratio scale. First, there is a true zero point; some subjects may get no items correct at all. Moreover, a difference of one represents a difference of one item recalled across the entire scale. It is certainly valid to say that someone who recalled 12 items recalled twice as many items as someone who recalled only 6 items.

But number-of-items recalled is a more complicated case than it appears at first. Consider the following example in which subjects are asked to remember as many items as possible from a list of 10. Assume that (a) there are 5 easy items and 5 difficult items, (b) half of the subjects are able to recall all the easy items and different numbers of difficult items, while (c) the other half of the subjects are unable to recall any of the difficult items but they do remember different numbers of easy items. Some sample data are shown below.

Subject	Easy Items					Difficult Items					Score
A	0	0	1	1	0	0	0	0	0	0	2
B	1	0	1	1	0	0	0	0	0	0	3
C	1	1	1	1	1	1	1	0	0	0	7
D	1	1	1	1	1	0	1	1	0	1	8

Let's compare (i) the difference between Subject A's score of 2 and Subject B's score of 3 and (ii) the difference between Subject C's score of 7 and Subject D's score of 8. The former difference is a difference of one easy item; the latter difference is a difference of one difficult item. Do these two differences necessarily signify the same difference in memory? We are inclined to respond “No” to this question since only a little more memory may be needed to retain the additional easy item whereas a lot more memory may be needed to retain the additional hard item. The general point is that it is often inappropriate to consider psychological measurement scales as either interval or ratio.

Consequences of level of measurement

Why are we so interested in the type of scale that measures a dependent variable? The crux of the matter is the relationship between the variable's level of measurement and the statistics that can be meaningfully computed with that variable. For example, consider a hypothetical study in which 5 children are asked to choose their favorite color from blue, red, yellow, green, and purple. The researcher codes the results as follows:

Color	Code
Blue	1
Red	2
Yellow	3
Green	4
Purple	5

This means that if a child said her favorite color was “Red,” then the choice was coded as “2,” if the child said her favorite color was “Purple,” then the response was coded as 5, and so forth. Consider the following hypothetical data:

Subject	Color	Code
1	Blue	1
2	Blue	1
3	Green	4
4	Green	4
5	Purple	5

Each code is a number, so nothing prevents us from computing the average code assigned to the children. The average happens to be 3, but you can see that it would be senseless to conclude that the average favorite color is yellow (the color with a code of 3). Such nonsense arises because favorite color is a nominal scale, and taking the average of its numerical labels is like counting the number of letters in the name of a snake to see how long the beast is.

Does it make sense to compute the mean of numbers measured on an ordinal scale? This is a difficult question, one that statisticians have debated for decades. The prevailing (but by no means unanimous) opinion of statisticians is that for almost all practical situations, the mean of an ordinally-measured variable is a meaningful statistic. However, there are extreme situations in which computing the mean of an ordinally-measured variable can be very misleading.

Distributions

by David M. Lane and Heidi Ziemer

Prerequisites

- Chapter 1: Variables

Learning Objectives

1. Define “distribution”
2. Interpret a frequency distribution
3. Distinguish between a frequency distribution and a probability distribution
4. Construct a grouped frequency distribution for a continuous variable
5. Identify the skew of a distribution
6. Identify bimodal, leptokurtic, and platykurtic distributions

Distributions of Discrete Variables

I recently purchased a bag of Plain M&M's. The M&M's were in six different colors. A quick count showed that there were 55 M&M's: 17 brown, 18 red, 7 yellow, 7 green, 2 blue, and 4 orange. These counts are shown below in Table 1.

Table 1. Frequencies in the Bag of M&M's

Color	Frequency
Brown	17
Red	18
Yellow	7
Green	7
Blue	2
Orange	4

This table is called a frequency table and it describes the distribution of M&M color frequencies. Not surprisingly, this kind of distribution is called a frequency distribution. Often a frequency distribution is shown graphically as in Figure 1.

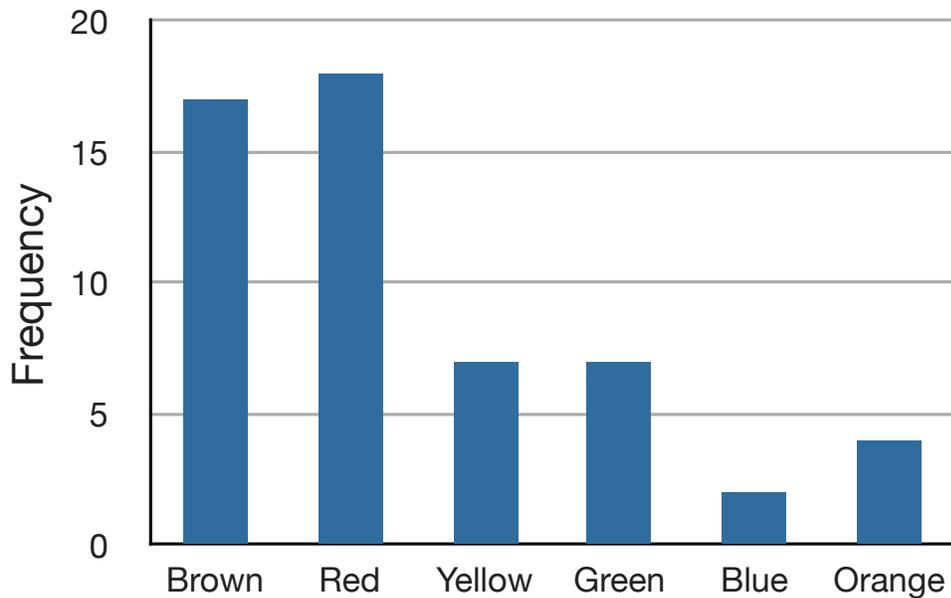


Figure 1. Distribution of 55 M&M's.

The distribution shown in Figure 1 concerns just my one bag of M&M's. You might be wondering about the distribution of colors for all M&M's. The manufacturer of M&M's provides some information about this matter, but they do not tell us exactly how many M&M's of each color they have ever produced. Instead, they report proportions rather than frequencies. Figure 2 shows these proportions. Since every M&M is one of the six familiar colors, the six proportions shown in the figure add to one. We call Figure 2 a probability distribution because if you choose an M&M at random, the probability of getting, say, a brown M&M is equal to the proportion of M&M's that are brown (0.30).

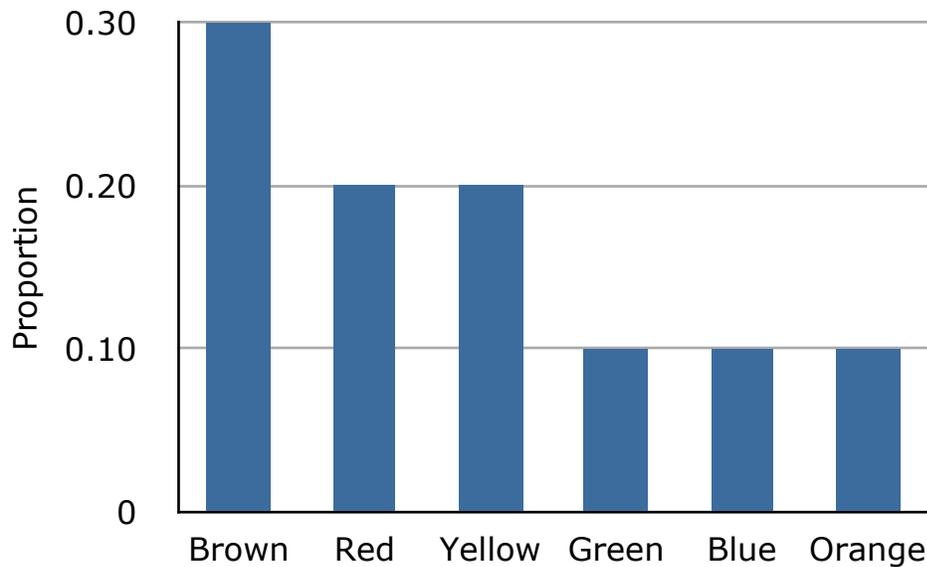


Figure 2. Distribution of all M&M's.

Notice that the distributions in Figures 1 and 2 are not identical. Figure 1 portrays the distribution in a sample of 55 M&M's. Figure 2 shows the proportions for all M&M's. Chance factors involving the machines used by the manufacturer introduce random variation into the different bags produced. Some bags will have a distribution of colors that is close to Figure 2; others will be further away.

Continuous Variables

The variable “color of M&M” used in this example is a discrete variable, and its distribution is also called discrete. Let us now extend the concept of a distribution to continuous variables.

The data shown in Table 2 are the times it took one of us (DL) to move the cursor over a small target in a series of 20 trials. The times are sorted from shortest to longest. The variable “time to respond” is a continuous variable. With time measured accurately (to many decimal places), no two response times would be expected to be the same. Measuring time in milliseconds (thousandths of a second) is often precise enough to approximate a continuous variable in psychology. As you can see in Table 2, measuring DL's responses this way produced times no two of which were the same. As a result, a frequency distribution would be uninformative: it would consist of the 20 times in the experiment, each with a frequency of 1.

Table 2. Response Times

568	720
577	728
581	729
640	777
641	808
645	824
657	825
673	865
696	875
703	1007

The solution to this problem is to create a grouped frequency distribution. In a grouped frequency distribution, scores falling within various ranges are tabulated. Table 3 shows a grouped frequency distribution for these 20 times.

Table 3. Grouped frequency distribution

Range	Frequency
500-600	3
600-700	6
700-800	5
800-900	5
900-1000	0
1000-1100	1

Grouped frequency distributions can be portrayed graphically. Figure 3 shows a graphical representation of the frequency distribution in Table 3. This kind of graph is called a histogram. Chapter 2 contains an entire section devoted to histograms.

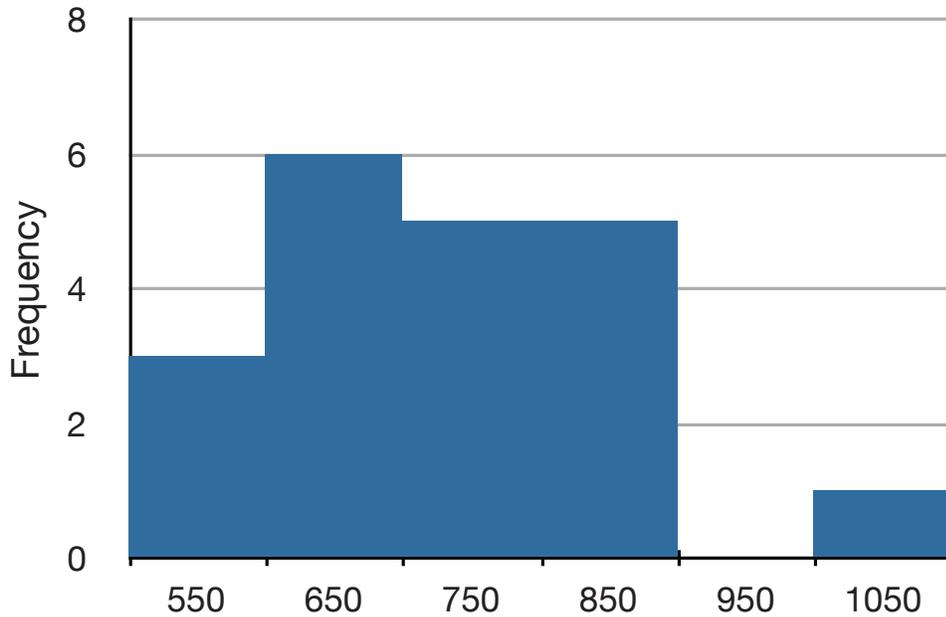


Figure 3. A histogram of the grouped frequency distribution shown in Table 3. The labels on the X-axis are the middle values of the range they represent.

Probability Densities

The histogram in Figure 3 portrays just DL's 20 times in the one experiment he performed. To represent the probability associated with an arbitrary movement (which can take any positive amount of time), we must represent all these potential times at once. For this purpose, we plot the distribution for the continuous variable of time. Distributions for continuous variables are called continuous distributions. They also carry the fancier name probability density. Some probability densities have particular importance in statistics. A very important one is shaped like a bell, and called the normal distribution. Many naturally-occurring phenomena can be approximated surprisingly well by this distribution. It will serve to illustrate some features of all continuous distributions.

An example of a normal distribution is shown in Figure 4. Do you see the “bell”? The normal distribution doesn't represent a real bell, however, since the left and right tips extend indefinitely (we can't draw them any further so they look like they've stopped in our diagram). The Y-axis in the normal distribution represents the “density of probability.” Intuitively, it shows the chance of obtaining values near corresponding points on the X-axis. In Figure 4, for example, the probability of an observation with value near 40 is about half of the probability of an

observation with value near 50. (For more information, see Chapter 7.)

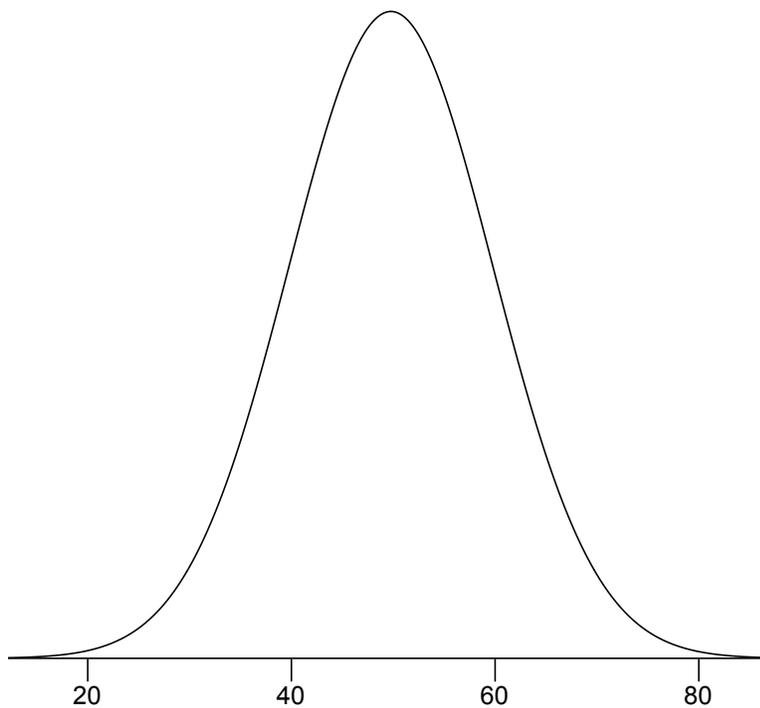


Figure 4. A normal distribution.

Although this text does not discuss the concept of probability density in detail, you should keep the following ideas in mind about the curve that describes a continuous distribution (like the normal distribution). First, the area under the curve equals 1. Second, the probability of any exact value of X is 0. Finally, the area under the curve and bounded between two given points on the X -axis is the probability that a number chosen at random will fall between the two points. Let us illustrate with DL's hand movements. First, the probability that his movement takes some amount of time is one! (We exclude the possibility of him never finishing his gesture.) Second, the probability that his movement takes exactly 598.956432342346576 milliseconds is essentially zero. (We can make the probability as close as we like to zero by making the time measurement more and more precise.) Finally, suppose that the probability of DL's movement taking between 600 and 700 milliseconds is one tenth. Then the continuous distribution for DL's possible times would have a shape that places 10% of the area below the curve in the region bounded by 600 and 700 on the X -axis.

Shapes of Distributions

Distributions have different shapes; they don't all look like the normal distribution in Figure 4. For example, the normal probability density is higher in the middle compared to its two tails. Other distributions need not have this feature. There is even variation among the distributions that we call “normal.” For example, some normal distributions are more spread out than the one shown in Figure 4 (their tails begin to hit the X-axis further from the middle of the curve --for example, at 10 and 90 if drawn in place of Figure 4). Others are less spread out (their tails might approach the X-axis at 30 and 70). More information on the normal distribution can be found in a later chapter completely devoted to them.

The distribution shown in Figure 4 is symmetric; if you folded it in the middle, the two sides would match perfectly. Figure 5 shows the discrete distribution of scores on a psychology test. This distribution is not symmetric: the tail in the positive direction extends further than the tail in the negative direction. A distribution with the longer tail extending in the positive direction is said to have a positive skew. It is also described as “skewed to the right.”

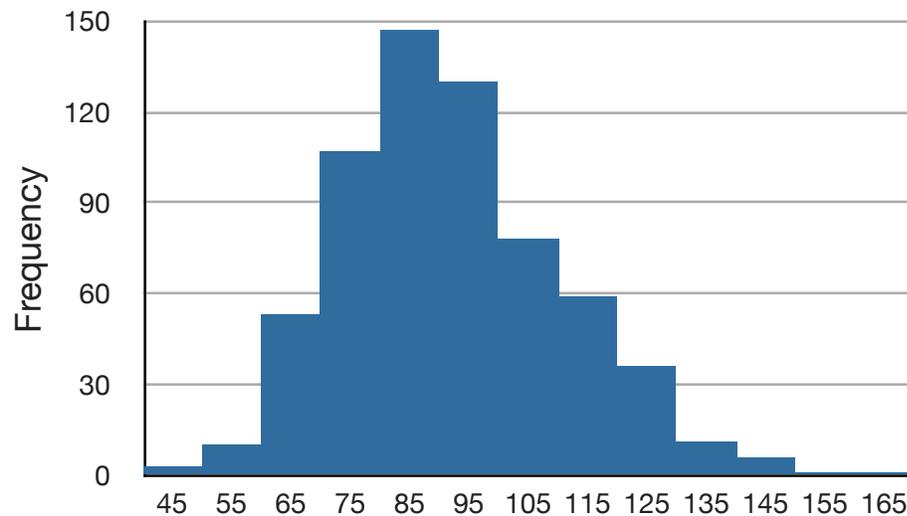


Figure 5. A distribution with a positive skew.

Figure 6 shows the salaries of major league baseball players in 1974 (in thousands of dollars). This distribution has an extreme positive skew.

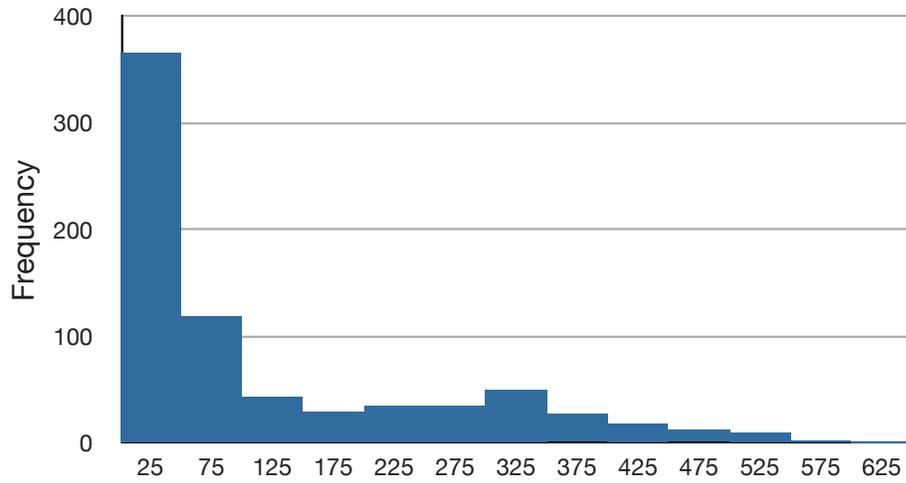


Figure 6. A distribution with a very large positive skew.

A continuous distribution with a positive skew is shown in Figure 7.

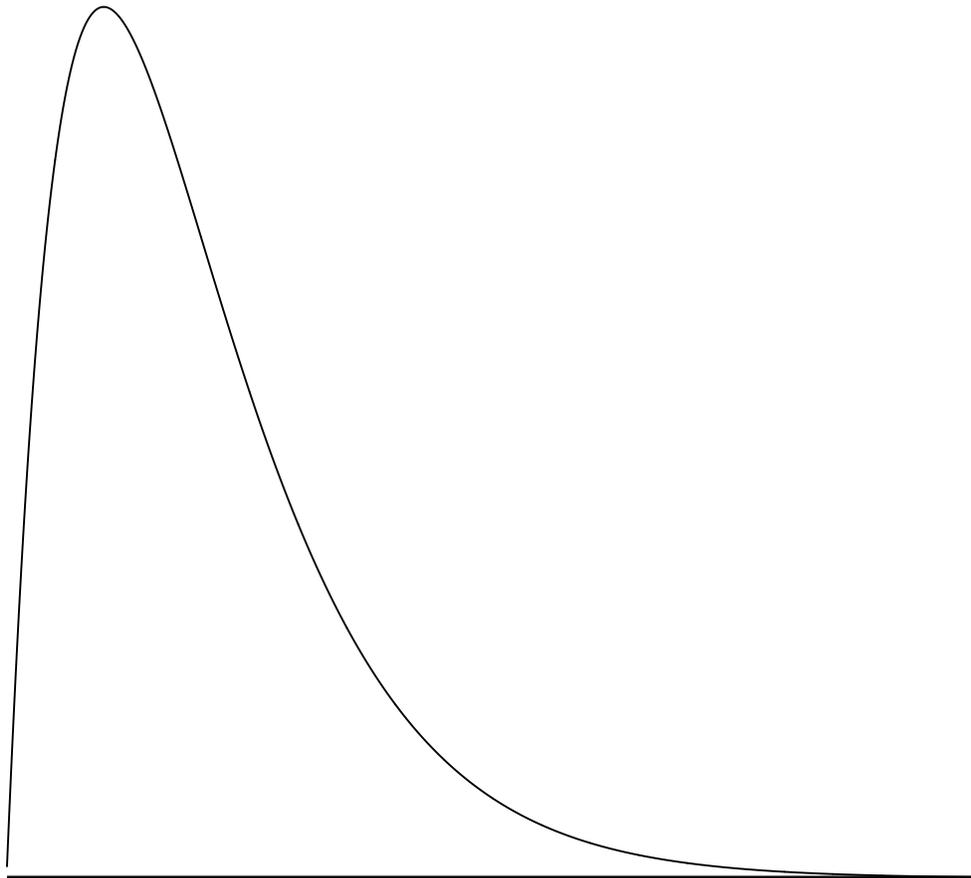


Figure 7. A continuous distribution with a positive skew.

Although less common, some distributions have a negative skew. Figure 8 shows the scores on a 20-point problem on a statistics exam. Since the tail of the distribution extends to the left, this distribution is skewed to the left.

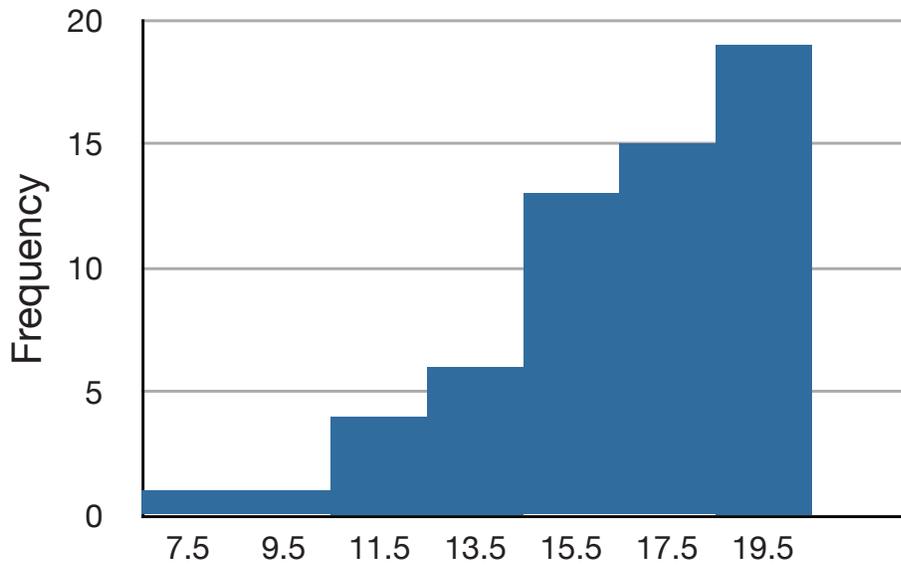


Figure 8. A distribution with negative skew. This histogram shows the frequencies of various scores on a 20-point question on a statistics test.

A continuous distribution with a negative skew is shown in Figure 9.

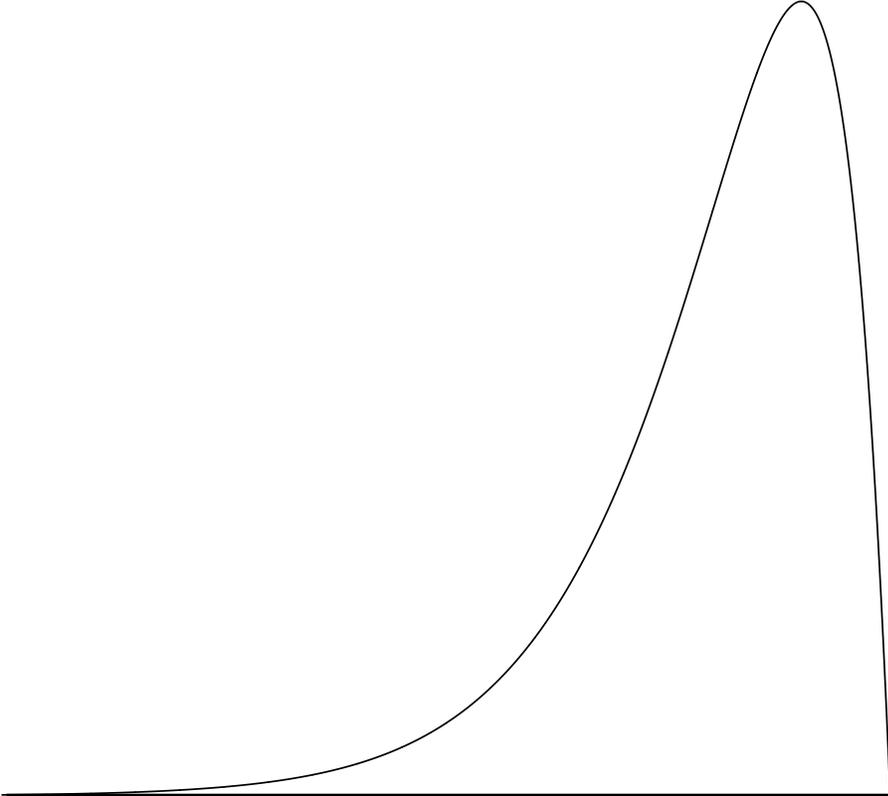


Figure 9. A continuous distribution with a negative skew.

The distributions shown so far all have one distinct high point or peak. The distribution in Figure 10 has two distinct peaks. A distribution with two peaks is called a bimodal distribution.

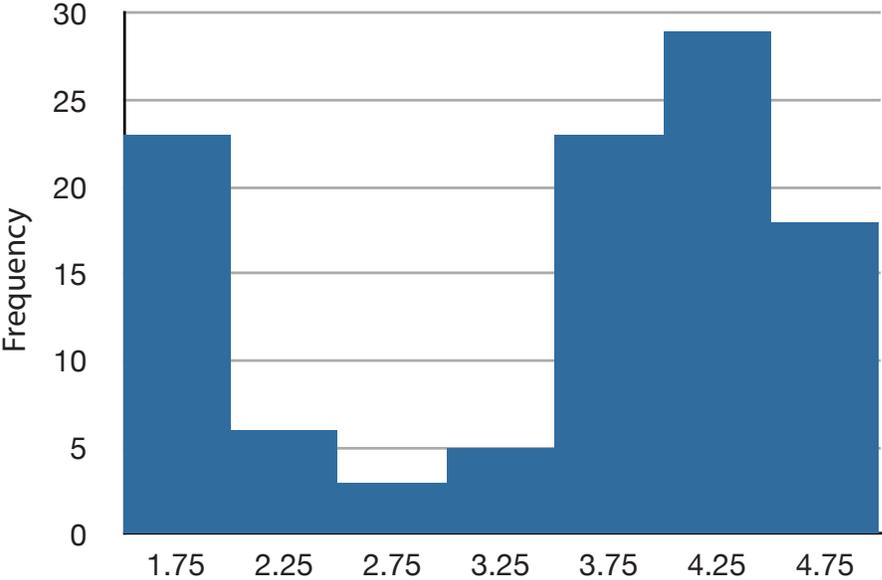


Figure 10. Frequencies of times between eruptions of the Old Faithful geyser. Notice the two distinct peaks: one at 1.75 and the other at 4.25.

Distributions also differ from each other in terms of how large or “fat” their tails are. Figure 11 shows two distributions that differ in this respect. The upper distribution has relatively more scores in its tails; its shape is called leptokurtic. The lower distribution has relatively fewer scores in its tails; its shape is called platykurtic.

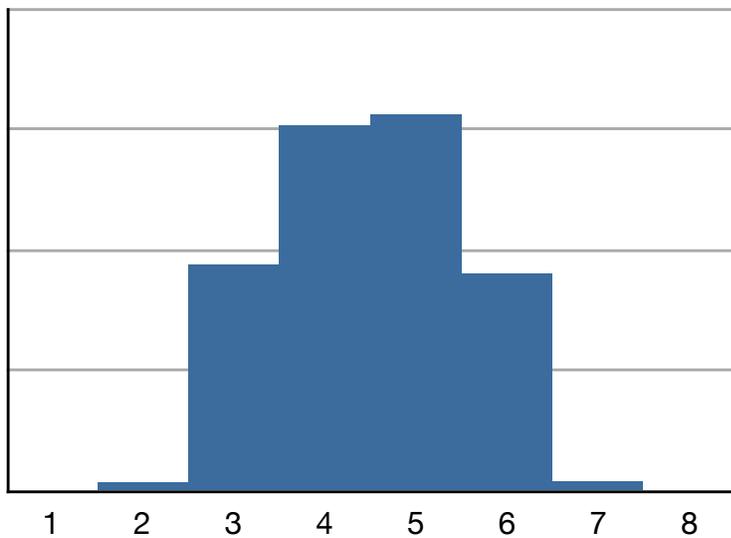
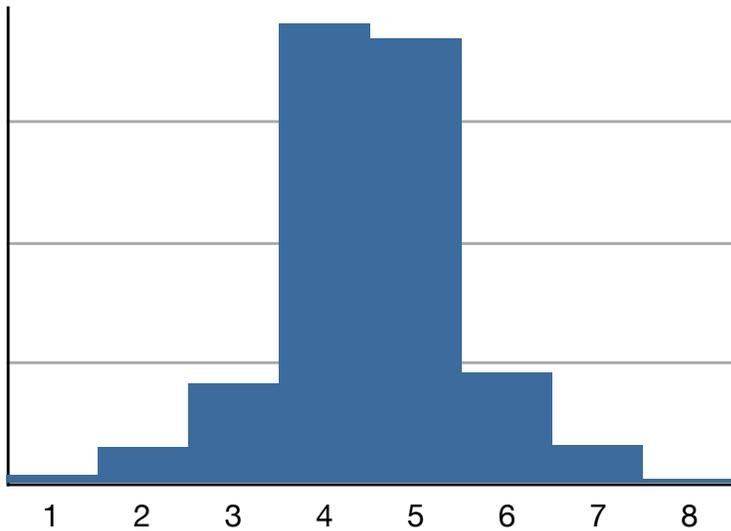


Figure 11. Distributions differing in kurtosis. The top distribution has long tails. It is called “leptokurtic.” The bottom distribution has short tails. It is called “platykurtic.”

Summation Notation

by David M. Lane

Prerequisites

- None

Learning Objectives

1. Use summation notation to express the sum of all numbers
2. Use summation notation to express the sum of a subset of numbers
3. Use summation notation to express the sum of squares

Many statistical formulas involve summing numbers. Fortunately there is a convenient notation for expressing summation. This section covers the basics of this summation notation.

Let's say we have a variable X that represents the weights (in grams) of 4 grapes. The data are shown in Table 1.

Table 1. Weights of 4 grapes.

Grape	X
1	4.6
2	5.1
3	4.9
4	4.4

We label Grape 1's weight X_1 , Grape 2's weight X_2 , etc. The following formula means to sum up the weights of the four grapes:

$$\sum_{i=1}^4 X_i$$

The Greek letter Σ indicates summation. The “ $i = 1$ ” at the bottom indicates that the summation is to start with X_1 and the 4 at the top indicates that the summation will end with X_4 . The “ X_i ” indicates that X is the variable to be summed as i goes from 1 to 4. Therefore,

$$\sum_{i=1}^4 X_i = X_1 + X_2 + X_3 + X_4 = 4.6 + 5.1 + 4.9 + 4.4 = 19$$

The symbol

$$\sum_{i=1}^3 X_i$$

indicates that only the first 3 scores are to be summed. The index variable i goes from 1 to 3.

When all the scores of a variable (such as X) are to be summed, it is often convenient to use the following abbreviated notation:

$$\sum X$$

Thus, when no values of i are shown, it means to sum all the values of X .

Many formulas involve squaring numbers before they are summed. This is indicated as

$$\begin{aligned} \sum X^2 &= 4.6^2 + 5.1^2 + 4.9^2 + 4.4^2 \\ &= 21.16 + 26.01 + 24.01 + 19.36 = 90.54. \end{aligned}$$

Notice that:

$$\left(\sum X\right)^2 \neq \sum X^2$$

because the expression on the left means to sum up all the values of X and then square the sum ($19^2 = 361$), whereas the expression on the right means to square the numbers and then sum the squares (90.54, as shown).

Some formulas involve the sum of cross products. Table 2 shows the data for variables X and Y . The cross products (XY) are shown in the third column. The sum of the cross products is $3 + 4 + 21 = 28$.

Table 2. Cross Products.

X	Y	XY
1	3	3
2	2	4
3	7	21

In summation notation, this is written as:

$$\sum XY = 28$$

Linear Transformations

by David M. Lane

Prerequisites

- None

Learning Objectives

1. Give the formula for a linear transformation
2. Determine whether a transformation is linear
3. Describe what is linear about a linear transformation

Often it is necessary to transform data from one measurement scale to another. For example, you might want to convert height measured in feet to height measured in inches. Table 1 shows the heights of four people measured in both feet and inches. To transform feet to inches, you simply multiply by 12. Similarly, to transform inches to feet, you divide by 12.

Table 1. Converting between feet and inches.

Feet	Inches
5.00	60
6.25	75
5.50	66
5.75	69

Some conversions require that you multiply by a number and then add a second number. A good example of this is the transformation between degrees Centigrade and degrees Fahrenheit. Table 2 shows the temperatures of 5 US cities in the early afternoon of November 16, 2002.

Table 2. Temperatures in 5 cities on 11/16/2002.

City	Degrees Fahrenheit	Degrees Centigrade
Houston	54	12.22
Chicago	37	2.78
Minneapolis	31	-0.56
Miami	78	25.56
Phoenix	70	21.11

The formula to transform Centigrade to Fahrenheit is:

$$F = 1.8C + 32$$

The formula for converting from Fahrenheit to Centigrade is

$$C = 0.5556F - 17.778$$

The transformation consists of multiplying by a constant and then adding a second constant. For the conversion from Centigrade to Fahrenheit, the first constant is 1.8 and the second is 32.

Figure 1 shows a plot of degrees Centigrade as a function of degrees Fahrenheit. Notice that the points form a straight line. This will always be the case if the transformation from one scale to another consists of multiplying by one constant and then adding a second constant. Such transformations are therefore called linear transformations.

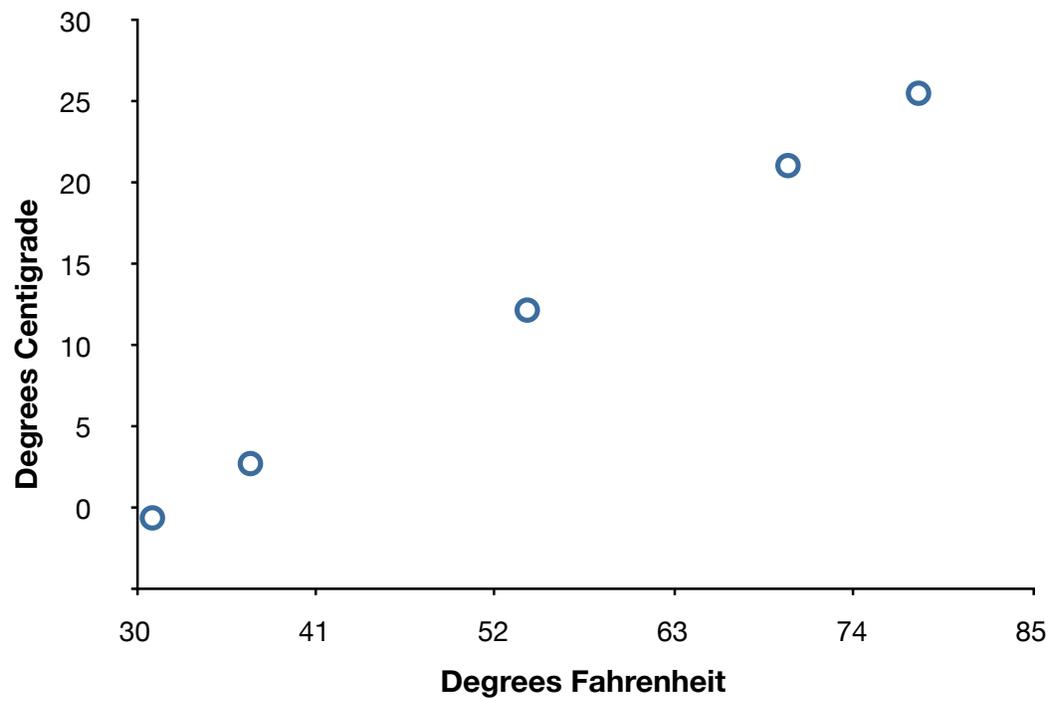


Figure 1. Degrees Centigrade as a function of degrees Fahrenheit

Logarithms

by David M. Lane

Prerequisites

- Chapter 1: Distributions

Learning Objectives

1. Compute logs using different bases
2. Convert between bases
3. State the relationship between logs and proportional change

The log transformation reduces positive skew. This can be valuable both for making the data more interpretable and for helping to meet the assumptions of inferential statistics.

Basics of Logarithms (Logs)

Logs are, in a sense, the opposite of exponents. Consider the following simple expression:

$$10^2 = 100$$

Here we can say the base of 10 is raised to the second power. Here is an example of a log:

$$\text{Log}_{10}(100) = 2$$

This can be read as: The log base ten of 100 equals 2. The result is the power that the base of 10 has to be raised to in order to equal the value (100). Similarly,

$$\text{Log}_{10}(1000) = 3$$

since 10 has to be raised to the third power in order to equal 1,000.

These examples all used base 10, but any base could have been used. There is a base which results in “natural logarithms” and that is called e and equals approximately 2.718. It is beyond the scope of this book to explain what is “natural” about it. Natural logarithms can be indicated either as: $\text{Ln}(x)$ or $\text{log}_e(x)$

Changing the base of the log changes the result by a multiplicative constant. To convert from Log_{10} to natural logs, you multiply by 2.303. Analogously, to convert in the other direction, you divide by 2.303.

Taking the antilog of a number undoes the operation of taking the log. Therefore, since $\text{Log}_{10}(1000) = 3$, the antilog_{10} of 3 is 1,000. Taking the antilog of a number simply raises the base of the logarithm in question to that number.

Logs and Proportional Change

A series of numbers that increases proportionally will increase in equal amounts when converted to logs. For example, the numbers in the first column of Table 1 increase by a factor of 1.5 so that each row is 1.5 times as high as the preceding row. The Log_{10} transformed numbers increase in equal steps of 0.176.

Table 1. Proportional raw changes are equal in log units.

Raw	Log
4.0	0.602
6.0	0.778
9.0	0.954
13.5	1.130

As another example, if one student increased their score from 100 to 200 while a second student increased their's from 150 to 300, the percentage change (100%) is the same for both students. The log difference is also the same, as shown below.

$$\begin{aligned}\text{Log}_{10}(100) &= 2.000 \\ \text{Log}_{10}(200) &= 2.301 \\ \text{Difference:} &= 0.301\end{aligned}$$

$$\begin{aligned}\text{Log}_{10}(150) &= 2.176 \\ \text{Log}_{10}(300) &= 2.477 \\ \text{Difference:} &= 0.301\end{aligned}$$

Arithmetic Operations

Rules for logs of products and quotients are shown below.

$$\text{Log}(AB) = \text{Log}(A) + \text{Log}(B)$$

$$\text{Log}(A/B) = \text{Log}(A) - \text{Log}(B)$$

For example,

$$\text{Log}_{10}(10 \times 100) = \text{Log}_{10}(10) + \text{Log}_{10}(100) = 1 + 2 = 3.$$

Similarly,

$$\text{Log}_{10}(100/10) = \text{Log}_{10}(100) - \text{Log}_{10}(10) = 2 - 1 = 1.$$

Statistical Literacy

by Denise Harvey and David M. Lane

Prerequisites

- Chapter 1: Levels of Measurement

The Board of Trustees at a university commissioned a top management-consulting firm to address the admission processes for academic and athletic programs. The consulting firm wrote a report discussing the trade-off between maintaining academic and athletic excellence. One of their key findings was:

The standard for an athlete's admission, as reflected in SAT scores alone, is lower than the standard for non-athletes by as much as 20 percent, with the weight of this difference being carried by the so-called "revenue sports" of football and basketball. Athletes are also admitted through a different process than the one used to admit non-athlete students.

What do you think?

Based on what you have learned in this chapter about measurement scales, does it make sense to compare SAT scores using percentages? Why or why not?

Think about this before continuing:

As you may know, the SAT has an arbitrarily-determined lower limit on test scores of 200. Therefore, SAT is measured on either an ordinal scale or, at most, an interval scale. However, it is clearly not measured on a ratio scale. Therefore, it is not meaningful to report SAT score differences in terms of percentages. For example, consider the effect of subtracting 200 from every student's score so that the lowest possible score is 0. How would that affect the difference as expressed in percentages?

Exercises

Prerequisites

- All material presented in Chapter: “Introduction”
1. A teacher wishes to know whether the males in his/her class have more conservative attitudes than the females. A questionnaire is distributed assessing attitudes and the males and the females are compared. Is this an example of descriptive or inferential statistics?
 2. A cognitive psychologist is interested in comparing two ways of presenting stimuli on sub-sequent memory. Twelve subjects are presented with each method and a memory test is given. What would be the roles of descriptive and inferential statistics in the analysis of these data?
 3. If you are told only that you scored in the 80th percentile, do you know from that description exactly how it was calculated? Explain.
 4. A study is conducted to determine whether people learn better with spaced or massed practice. Subjects volunteer from an introductory psychology class. At the beginning of the semester 12 subjects volunteer and are assigned to the massed-practice condition. At the end of the semester 12 subjects volunteer and are assigned to the spaced-practice condition. This experiment involves two kinds of non-random sampling: (1) Subjects are not randomly sampled from some specified population and (2) subjects are not randomly assigned to conditions. Which of the problems relates to the generality of the results? Which of the problems relates to the validity of the results? Which problem is more serious?
 5. Give an example of an independent and a dependent variable.
 6. Categorize the following variables as being qualitative or quantitative:
 7. Specify the level of measurement used for the items in Question 6.
 8. Which of the following are linear transformations?

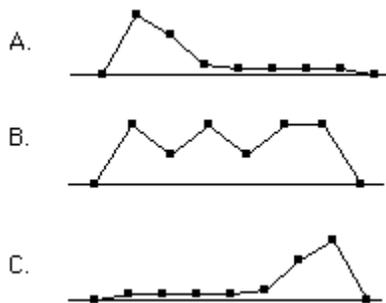
9. The formula for finding each student's test grade (g) from his or her raw score (s) on a test is as follows: $g = 16 + 3s$

Is this a linear transformation?

If a student got a raw score of 20, what is his test grade?

10. For the numbers 1, 2, 4, 16, compute the following:

11. Which of the frequency polygons has a large positive skew? Which has a large negative skew?



12. What is more likely to have a skewed distribution: time to solve an anagram problem (where the letters of a word or phrase are rearranged into another word or phrase like “dear” and “read” or “funeral” and “real fun”) or scores on a vocabulary test?

Questions from Case Studies

Angry Moods (AM) case study

13. (AM) Which variables are the participant variables? (They act as independent variables in this study.)

14. (AM) What are the dependent variables?

15. (AM) Is Anger-Out a quantitative or qualitative variable?

Teacher Ratings (TR) case study

16. (TR) What is the independent variable in this study?

ADHD Treatment (AT) case study

17. (AT) What is the independent variable of this experiment? How many levels does it have?

18. (AT) What is the dependent variable? On what scale (nominal, ordinal, interval, ratio) was it measured?