Chapter 5 Exercises

Prerequisites
All material presented in chapter 5

Selected answers

You may want to use the Binomial Calculator for some of these exercises.

1. (a) What is the probability of rolling a pair of dice and obtaining a total score of 9 or more? (b) What is the probability of rolling a pair of dice and obtaining a total score of 7? (relevant section)
2. A box contains four black pieces of cloth, two striped pieces, and six dotted pieces. A piece is selected randomly and then placed back in the box. A second piece is selected randomly. What is the probability that:
   a. both pieces are dotted?
   b. the first piece is black and the second piece is dotted?
   c. one piece is black and one piece is striped?
      (relevant section)
3. A card is drawn at random from a deck. (a) What is the probability that it is an ace or a king? (b) What is the probability that it is either a red card or a black card? (relevant section)
4. The probability that you will win a game is 0.45. (a) If you play the game 80 times, how many times would you expect to win? (b) What are the mean and standard deviation of a binomial distribution with $\pi = 0.45$ and $N = 80$? (relevant section)
5. A fair coin is flipped 9 times. What is the probability of getting exactly 6 heads? (relevant section)
6. When Susan and Jessica play a card game, Susan wins 60% of the time. If they play 9 games, what is the probability that Jessica will have won more games than Susan? (relevant section)
7. You flip a coin three times. (a) What is the probability of getting heads on only one of your flips? (b) What is the probability of getting heads on at least one flip? (relevant section & relevant section)
8. A test correctly identifies a disease in 95% of people who have it. It correctly identifies no disease in 94% of people who do not have it. In the population, 3% of the people have the disease. What is the probability that you have the disease if you tested positive? (relevant section)
9. A jar contains 10 blue marbles, 5 red marbles, 4 green marbles, and 1 yellow marble. Two marbles are chosen (without replacement). (a) What is the probability that one will be green and the other red? (b) What is the probability that one will be blue and the other yellow? (relevant section)

10. You roll a fair die five times, and you get a 6 each time. What is the probability that you get a 6 on the next roll? (relevant section)
11. You win a game if you roll a die and get a 2 or a 5. You play this game 60 times.
    a. What is the probability that you win between 5 and 10 times (inclusive)?
    b. What is the probability that you will win the game at least 15 times?
    c. What is the probability that you will win the game at least 40 times?
    d. What is the most likely number of wins.
    e. What is the probability of obtaining the number of wins in d? (relevant section)
12. In a baseball game, Tommy gets a hit 30% of the time when facing this pitcher. Joey gets a hit 25% of the time. They are both coming up to bat this inning.
    a. What is the probability that Joey or Tommy will get a hit?
    b. What is the probability that neither player gets a hit?
    c. What is the probability that they both get a hit? (relevant section)
13. An unfair coin has a probability of coming up heads of 0.65. The coin is flipped 50 times. What is the probability it will come up heads 25 or fewer times? (Give answer to at least 3 decimal places). (relevant section)
14. You draw two cards from a deck, what is the probability that
    a. both of them are face cards (king, queen, or jack)?
    b. What is the probability that you draw two cards from a deck and both of them are hearts? (relevant section)
15. True/False: You are more likely to get a pattern of HTHHHTHTTH than HHHHHHHHTT when you flip a coin 10 times. (relevant section)
16. True/False: Suppose that at your regular physical exam you test positive for a relatively rare disease. You will need to start taking medicine if you have the disease, so you ask your doctor about the accuracy of the test. It turns out that the test is 98% accurate. The probability that you have Disease X is therefore 0.98 and the probability that you do not have it is .02. (relevant section)

Questions from Case Studies:

The following questions are from the Diet and Health (DH) case study.
17. (DH#1)
    a. What percentage of people on the AHA diet had some sort of illness or death?
    b. What is the probability that if you randomly selected a person on the AHA diet, he or she would have some sort of illness or death?

(relevant section)

   c. If 3 people on the AHA diet are chosen at random, what is the probability that they will all be healthy? (relevant section)

18. (DH#2)

   a. What percentage of people on the Mediterranean diet had some sort of illness or death?

   b. What is the probability that if you randomly selected a person on the Mediterranean diet, he or she would have some sort of illness or death? (relevant section)

   c. What is the probability that if you randomly selected a person on the Mediterranean diet, he or she would have cancer? (relevant section)

   d. If you randomly select five people from the Mediterranean diet, what is the probability that they would all be healthy? (relevant section)

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19. Five faces of a fair die are painted black, and one face is painted white. The die is rolled six times. Which of the following results is more likely?
a. Black side up on five of the rolls; white side up on the other roll
b. Black side up on all six rolls
c. a and b are equally likely

20. One of the items on the student survey for an introductory statistics course was "Rate your intelligence on a scale of 1 to 10." The distribution of this variable for the 100 women in the class is presented below. What is the probability of randomly selecting a women from the class who has an intelligence rating that is LESS than seven (7)?

Intelligence Rating	Count
5	12
6	24
7	38
8	23
9	2
10	1

a. (12 + 24)/100 = .36

b. (12 + 24 + 38)/100 = .74

c. 38/100 = .38

d. (23 + 2 + 1)/100 = .26

e. None of the above.

21. If the occurrence of one event does not influence the outcome of another event, then two events are:
a. conditional

b. disjoint

c. independent

d. interdependent

22. You roll 2 fair six-sided dice. Which of the following outcomes is most likely to occur on the next roll? A. Getting double 3. B. Getting a 3 and a 4. C. They are equally likely. Explain your choice.

23. If Tahnee flips a coin 10 times, and records the results (Heads or Tails), which outcome below is more likely to occur, A or B? Explain your choice.

Throw Number	1	2	3	4	5	6	7	8	9	10
A:	H	T	T	H	T	H	H	T	T	T
B:	H	T	H	T	H	T	H	T	H	T

24. A bowl has 100 wrapped hard candies in it. 20 are yellow, 50 are red, and 30 are blue. They are well mixed up in the bowl. Jenny pulls out a handful

of 10 candies, counts the number of reds, and tells her teacher. The teacher writes the number of red candies on a list. Then, Jenny puts the candies back into the bowl, and mixes them all up again. Four of Jenny's classmates, Jack, Julie, Jason, and Jerry do the same thing. They each pick ten candies, count the reds, and the teacher writes down the number of reds. Then they put the candies back and mix them up again each time. The teacher's list for the number of reds is most likely to be (please select one):

a. 8,9,7,10,9
b. 3,7,5,8,5
c. 5,5,5,5,5
d. 2,4,3,4,3
e. 3,0,9,2,8

25. An insurance company writes policies for a large number of newly-licensed drivers each year. Suppose 40% of these are low-risk drivers, 40% are moderate risk, and 20% are high risk. The company has no way to know which group any individual driver falls in when it writes the policies. None of the low-risk drivers will have an at-fault accident in the next year, but 10% of the moderate-risk and 20% of the high-risk drivers will have such an accident. If a driver has an at-fault accident in the next year, what is the probability that he or she is high-risk?

26. You are to participate in an exam for which you had no chance to study, and for that reason cannot do but guess for each question (all questions being of the multiple choice type, so the chance of guessing the correct answer for each question is 1/d, d being the number of options (distractors) per question; so in case of a 4-choice question, your guess chance is 0.25). Your instructor offers you the opportunity to choose amongst the following exam formats: I. 6 questions of the 4-choice type; you pass when 5 or more answers are correct; II. 5 questions of the 5-choice type; you pass when 4 or more answers are correct; III. 4 questions of the 10-choice type; you pass when 3 or more answers are correct. Rank the three exam formats according to their attractiveness. It should be clear that the format with the highest probability to pass is the most attractive format. Which would you choose and why?

27. Consider the question of whether the home team wins more than half of its games in the National Basketball Association. Suppose that you study a simple random sample of 80 professional basketball games and find that 52 of them are won by the home team.

   a. Assuming that there is no home court advantage and that the home team therefore wins 50% of its games in the long run, determine the probability that the home team would win 65% or more of its games in a simple random sample of 80 games.

    b. Does the sample information (that 52 of a random sample of 80 games are won by the home team) provide strong evidence that the home team wins more than half of its games in the long run? Explain.

28. A refrigerator contains 6 apples, 5 oranges, 10 bananas, 3 pears, 7 peaches, 11 plums, and 2 mangos.

    a. Imagine you stick your hand in this refrigerator and pull out a piece of fruit at random. What is the probability that you will pull out a pear?

    b. Imagine now that you put your hand in the refrigerator and pull out a piece of fruit. You decide you do not want to eat that fruit so you put it back into the refrigerator and pull out another piece of fruit. What is the probability that the first piece of fruit you pull out is a banana and the second piece you pull out is an apple?

    c. What is the probability that you stick your hand in the refrigerator one time and pull out a mango or an orange?

Answers:

1) (a) 5/18 = .278

2) (b) 1/6 = .167

3) (a) 2/13 = .154

4) (b) 4.45

5) .164

7) (b) 7/8 = .875

9) (a) 2/19 = .105

11) (b) 20

12) (c) .075

14) (b) 1/17 = .0588

17) (c) .493

18) (b) .10