Effects of Linear Transformations

Prerequisites
Linear Transformations

This section covers the effects of linear transformations on measures of central tendency and variability. Let's start with an example we saw before in the section that defined linear transformation: temperatures of cities. Table 1shows the temperatures of 5 cities.

Table 1. Temperatures in 5 cities on 11/16/2002

City	Degrees Fahrenheit	Degrees Centigrade
Houston	54	12.22
Chicago	37	2.78
Minneapolis	31	-0.56
Miami	78	25.56
Phoenix	70	21.11
Mean	54.000	12.22
Median	54.000	12.22
Variance	330.00	101.852
SD	18.166	10.092

Recall that to transform the degrees Fahrenheit to degrees Centigrade, we use the formula

$$C = 0.55556F - 17.7778$$

which means we multiply each temperature Fahrenheit by 0.55556 and then subtract -17.778. As you might have expected, you multiply the mean temperature in Fahrenheit by 0.55556 and then subtract -17.778 to get the mean in Centigrade. That is, (0.55556)(54) - 17.7778 = 12.222. The same is true for the median. Note that this relationship holds even if the mean and median are not identical as they are in Table 1.

The formula for the standard deviation is just as simple: the standard deviation of degrees Centigrade is equal to the standard deviation in degrees Fahrenheit times 0.55556. Since the variance is the standard deviation squared, the variance in degrees Centigrade is equal to 0.55556^2 times the variance of degrees Fahrenheit.

To sum up, if a variable X has a mean of μ, a standard deviation of σ, and a variance of σ^2, then a new variable Y created using the linear transformation

$$Y = bX + A$$

will have a mean of $b\mu+A$, a standard deviation of $b\sigma$, and a variance of $b^2\sigma^2$.