Standard Normal Distribution

Prerequisites
Effects of Linear Transformations, Introduction to Normal Distributions

As discussed in the introductory section, normal distributions do not necessarily have the same means and standard deviations. A normal distribution with a mean of 0 and a standard deviation of 1 is called a *standard normal distribution*.

Areas of the normal distribution are often represented by tables of the standard normal distribution. A portion of a table of the standard normal distribution is shown in Table 1.

Table 1. A portion of a table of the standard normal distribution.

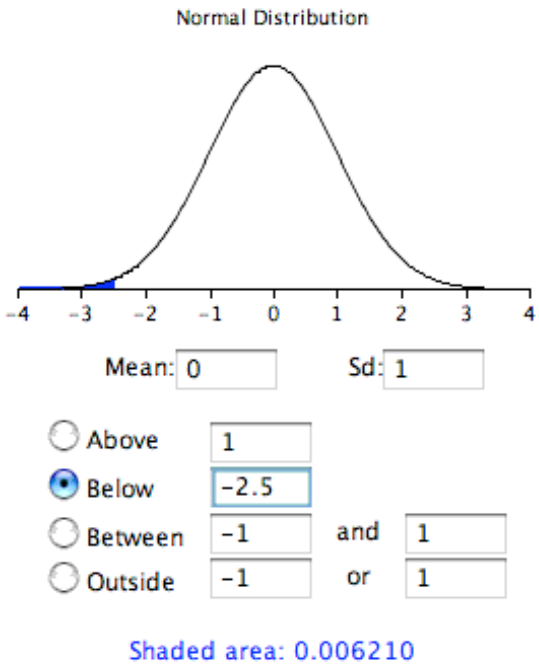
Z	Area below Z
-2.50	0.0062
-2.49	0.0064
-2.48	0.0066
-2.47	0.0068
-2.46	0.0069
-2.45	0.0071
-2.44	0.0073
-2.43	0.0075
-2.42	0.0078
-2.41	0.0080
-2.40	0.0082
-2.39	0.0084
-2.38	0.0087
-2.37	0.0089
-2.36	0.0091
-2.35	0.0094
-2.34	0.0096
-2.33	0.0099
-2.32	0.0102

The first column titled "Z" contains values of the standard normal distribution; the second column contains the area below Z. Since the distribution has a mean of 0 and a standard deviation of 1, the Z column is equal to the number of standard deviations below (or above) the mean. For example, a Z of -2.5 represents a value 2.5 standard deviations below the mean. The area below Z is

0.0062.

The same information can be obtained using the following Java applet. Figure 1 shows how it can be used to compute the area below a value of -2.5 on the standard normal distribution. Note that the mean is set to 0 and the standard deviation is set to 1.

Figure 1. An example from the applet.



Calculate Areas

A value from any normal distribution can be transformed into its corresponding value on a standard normal distribution using the following formula:

$$Z = (X - \mu)/\sigma$$

where Z is the value on the standard normal distribution, X is the value on the original distribution, μ is the mean of the original distribution and σ is the standard deviation of the original distribution.

As a simple application, what portion of a normal distribution with a mean of 50 and a standard deviation of 10 is below 26. Applying the formula we obtain

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Z = (26 - 50)/10 = -2.4.
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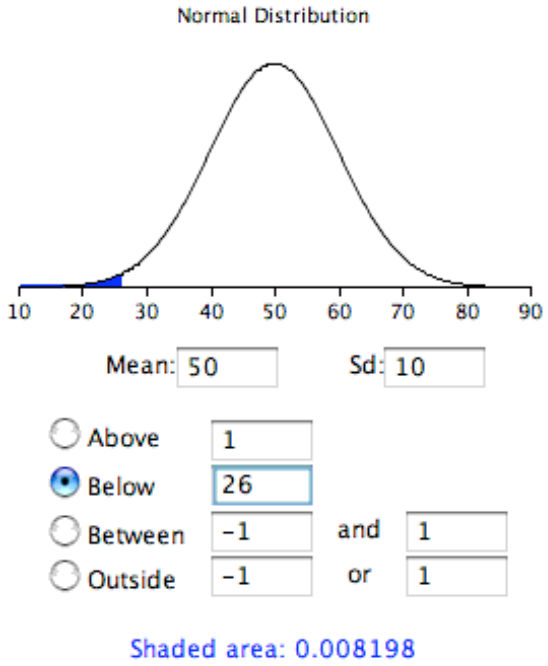
From Table 1, we can see that 0.0082 of the distribution is below -2.4. There is no need to transform to Z if you use the applet as shown in Figure 2.

Figure 2. Area below 36 in a normal distribution with a mean of 50 and a standard deviation of 10.



If all the values in a distribution are transformed to Z scores, then the distribution will have a mean of 0 and a standard distribution. This process of transforming a distribution to one with a mean of 0 and a standard deviation of 1 is called *standardizing* the distribution.